MASTERCLASS ABSTRACTS

Tara Brendle, Lecture 1

Title: The curve complex and duality for the mapping class group

Abstract: The complex of curves $C(S)$ associated to a surface S is a useful object in understanding just about every aspect of mapping class groups. For example, it plays the same role in the duality setting as the Tits building does for $\mathrm{SL}(n, \mathbb{Z})$. In our first lecture we will explore the structure of $C(S)$ through this lens, focusing first on its homotopy type.

Tara Brendle, Lecture 2

Title: The Steinberg module of the mapping class group

Abstract: We next turn our attention to homology of the curve complex $C(S)$; its 'top' homology is the Steinberg module for mapping class group. Our focus here will be Broaddus' result that the Steinberg module is cyclic as a mapping class group module, and the construction of a generating 'Broaddus sphere'.

Tara Brendle, Lecture 3

Title: Cohomology of congruence subgroups

Abstract: Congruence subgroups are an important family of finite index subgroups of mapping class groups. In our final session we will describe how the cohomology of congruence subgroups relates to that of the full mapping class group, and how to describe their rational cohomology in terms of the Steinberg module of the full mapping class group.

Kai-Uwe Bux, Lecture 1

Title: Outer space and its bordification

Abstract: I will introduce outer space as the moduli space of marked metric graphs of unit volume and fundamental group F_n; and I shall illustrate its relevance for the study of $\mathrm{Out}(F_n)$. Then I will turn to the bordification of outer space and how it enters the argument that $\mathrm{Out}(F_n)$ is a virtual duality group.

This talk is based upon work of others (in particular Karen Vogtmann and Marc Culler) and joint work with Peter Smillie and Karen Vogtmann. technical ingredient of the proof.

Kai-Uwe Bux, Lecture 2 + 3

Title: The dimension of the Torelli group for $\mathrm{Out}(F_n)$, part I + II

Abstract: The Torelli subgroup in the mapping class group of a surface consists of those mapping classes that act trivially in homology. Accordingly, the Torelli subgroup T_n of $\mathrm{Out}(F_n)$ consists of those outer automorphisms that act trivially on the abelianization of F_n.

The Torelli subgroup T_n is torsion free. It acts freely on the spine Y_n of outer space. As Y_n is contractible, the orbit space Y_n/T_n is a classifying space for T_n. I shall discuss, how one can use this to derive the cohomological dimension of T_n.

These sessions are based upon joint work with Mladen Bestvina and Dan Margalit.

Date: June 26, 2021.

Søren Galatius, Lecture 1-3

Title: High-dimensional cohomology of the moduli space of Riemann surfaces Part I-III

Abstract: My lectures will discuss the high dimensional cohomology of $\mathcal{M}_g$, the moduli space of genus g Riemann surfaces. With rational coefficients this agrees with the group cohomology of the mapping class group, which was proved by Harer to be a virtual duality group of dimension $4g-5$. After reviewing the abstract duality theory for this group, my lectures will focus on a connection to the cohomology of certain graph complexes, introduced by Kontsevich in the 1990s. Cohomology in degre $4g-6$ is particularly interesting, and turns out to have rather large dimension. My lectures will be based on joint work with Chan and Payne (arXiv: 1805.10186 and 1903.07187).

Jennifer Wilson, Lecture 1

Title: The top-degree rational cohomology of the special linear group of a number ring

Abstract: Let $\mathrm{SL}_n(R)$ denote the special linear group of a number ring R. We will review a lesson from earlier in the workshop: the groups $\mathrm{SL}_n(R)$ are *virtual duality groups*, and their rational cohomology is governed by a $\mathbb{Q}[\mathrm{SL}_n(R)]$-module called the Steinberg representation. We will survey some known vanishing and non-vanishing results on the high-degree rational cohomology of $\mathrm{SL}_n(R)$. Then, we will use a result of Ash–Rudolph to prove a result originally due to Lee–Szczarba: when R is a Euclidean domain, $H^{vcd}(\mathrm{SL}_n(R);\mathbb{Q}) = 0$.

Jennifer Wilson, Lecture 2

Title: A generating set for the Steinberg module of a Euclidean domain

Abstract: Let R be a Euclidean domain. Ash–Rudolph proved that the associated Steinberg module has a well-behaved generating set by *apartment classes* satisfying a certain integrality condition. In the first talk, we used their result to show that $H^{vcd}(\mathrm{SL}_n(R);\mathbb{Q})$ vanishes. In this talk, we will see a simplified proof of Ash–Rudolph's theorem due to Church–Farb–Putman and Maazen.

Benjamin Brück

Title: Free factor complexes as boundary structures

Abstract: Outer space is a moduli space of marked metric graphs that is widely seen as the $\mathrm{Out}(F_n)$-analogue of symmetric spaces and Teichmüller space. However, it is not so clear what the best analogues of Tits buildings and the curve complex in the setting of $\mathrm{Out}(F_n)$ are. My talk will be about two candidates for such analogues: the free factor complex and the complex of free factor systems. I will firstly explain how both of these complexes describe (parts of) the simplicial boundary of Outer space and secondly use this boundary description to show that both complexes are highly connected.

This is based on joint work with Radhika Gupta.

Mikala Ørsnes Jansen, Lecture 1

Title: Borel–Serre duality

Abstract: In 1973, Borel and Serre proved that torsion free arithmetic groups are *duality groups*, i.e. they satisfy a version of Poincaré duality with some additional twisting of the coefficient module. In this talk, we will review this result in the special case of finite index torsion free subgroups of $\mathrm{SL}_n(\mathbb{Z})$; we introduce the necessary definitions and give an outline of the proof. We will spend some time motivating the construction of the *Borel–Serre compactification*, the main technical ingredient of the proof.

Mikala Ørsnes Jansen, Lecture 2

Title: The stratified homotopy type of the reductive Borel–Serre compactification

Abstract: For neat arithmetic groups $\Gamma \leq \mathrm{SL}_n(\mathbb{Z})$, the locally symmetric space X associated with Γ provides a nice model for the classifying space $\mathrm{B}\,\Gamma$. Unfortunately, it is very rarely compact. To remedy this, various compactifications of X have been introduced. Motivated by an interest in L_2-cohomology, Zucker defined what is now known as the *reductive Borel-Serre compactification* in 1982. It comes equipped with a natural stratification, and we set out to understand this stratified space by determining its stratified homotopy type, more precisely its *exit path ∞-category.* This is an analogue for stratified spaces of the fundamental ∞-groupoid for topological spaces. A possible application is the study of cohomology theories with non-constant coefficients given by *constructible sheaves* as these are classified by the exit path ∞-category.

Jeremy Miller

Title: On not the dualizing module of $\mathrm{Aut}(F_n)$

Abstract: Bestvina-Feighn proved that $\mathrm{Aut}(F_n)$ is a virtual duality group but were not able to give a combinatorial description of the dualizing module. Motivated by the work of Borel–Serre on dualizing modules of arithmetic groups, Hatcher–Vogtmann asked if the top homology of the complex of free factors is the dualizing module of $\mathrm{Aut}(F_n)$. In joint work with Himes and Nariman, we show this is not the case.

Peter Patzt

Title: Introduction to the masterclass

Abstract: I will give a short overview of some of the directions this masterclass will take us. This includes moduli spaces, their connections to group cohomology, a duality property, and a conjecture of Church–Farb–Putman.

Robin Sroka

Title: On the high-dimensional rational cohomology of symplectic groups.

Abstract: Gunnells proved that the Steinberg module of the symplectic group is generated by integral apartment classes. This implies that the rational cohomology of the symplectic group $Sp_{2n}(\mathbb{Z})$ vanishes in its virtual cohomological dimension $v_n = n^2$. In 2020, Putman posed the problem of finding a Bykovskii-type presentation for the symplectic Steinberg module. The problem is closely related to the question whether the rational cohomology of the symplectic group vanishes in codimension-one, $v_n - 1$. This talk is based on joint work with Benjamin Brück and Peter Patzt. We will present an alternative proof of Gunnells' theorem following an idea of Putman and, if time permits, discuss work-in-progress on the rational codimension-one cohomology of symplectic groups.