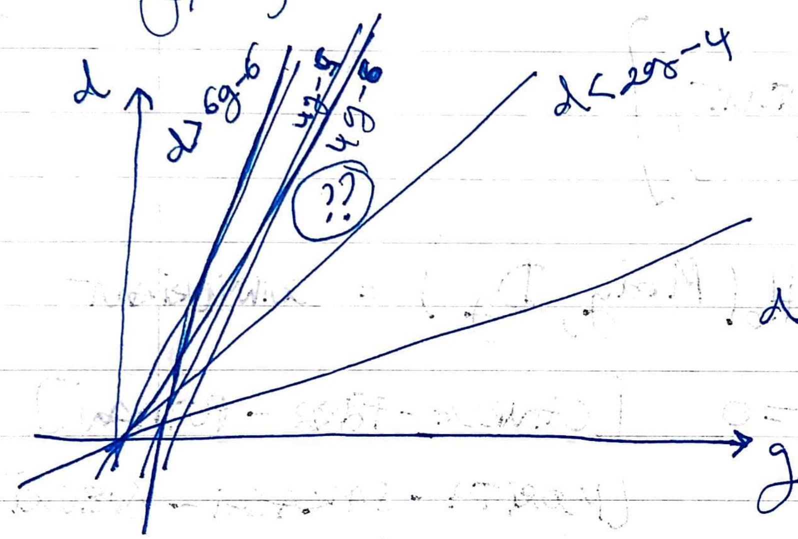
m 

## SØREN — HIGH DIM COH. OF $\mathcal{M}_{g,(n)}$

$\mathcal{M}_g$ = SPACE OF GENUS $g$ RIEM. SURFACES $/\sim$

$\cong$ SPACE OF GENUS $g$ ORIENT. 2-MFDS WITH HYPERBOLIC METRIC $/\sim$ $(g \geq 2)$

$H^d(\mathcal{M}_g; \mathbb{Q})$ "PHASE TRANSITIONS"




$\dim_{\mathbb{R}}(\mathcal{M}_g) = 6g-6 \Rightarrow H^d(\mathcal{M}_g; \mathcal{F}) = 0 \quad d > 6g-6$

"GEOMETRIC DIMENSION" $\mathcal{F}$ SHEAF

$\kappa_i \in H^{2i}(\mathcal{M}_g; \mathbb{Q})$ MILLER-MORITA-MUMFORD

$\mathbb{Q}[\kappa_1, \kappa_2, \ldots] \longrightarrow H^d(\mathcal{M}_g, \mathbb{Q})$ iso $d < \frac{g}{1.5}$

NONZERO in deg $2g-4$ ($\kappa_{g-2} \mapsto$ NON-ZERO)

RANK 1

VANISHES ABOVE

$\mathcal{M}_g = \mathcal{T}_g / \mathrm{Mod}_g$ $\qquad H^*(\mathcal{M}_g, \mathbb{Q}) = H^*(\mathrm{Mod}_g, \mathbb{Q})$

[HARER] $\mathrm{Mod}_g$ IS A VIRTUAL DUALITY GP, $\mathrm{vcd} = 4g-5$

$D_g = \tilde{H}_{g-2}(\mathcal{C}(S_g))$ DUALIZING MODULE.

$$H^i(\mathrm{Mod}_g ; M) \xrightarrow{\cong} H_{4g-5-i}(\mathrm{Mod}_g ; D_g \otimes_{\mathbb{Q}} M)$$

$M$: $\mathbb{Q}[\mathrm{Mod}_g]$-MODULE

$= 0$ FOR $i > 4g-5$

$\| \; [H^i(\mathcal{M}_g ; \text{LOCALLY CONSTANT SHEAF})]$

$$H^{4g-5}(\mathcal{M}_g ; \mathbb{Q}) \cong H_0(\mathrm{Mod}_g ; D_g) = \text{COINVARIANTS}$$

$= 0$ [CHURCH-FARB-PUTMAN]

[MORITA-SAKASAI-SUZUKI]

(HARER)

$\rightsquigarrow$ HIGHEST ~~[illegible]~~ DIMENSIONAL COHOMOLOGY POSSIBLY NON-ZERO

$$H^{4g-6}(\mathcal{M}_g ; \mathbb{Q}) = H_1(\mathrm{Mod}_g ; D_g)$$

ORBIFOLD $\|$ PD

$H_{2g}(\mathcal{M}_g ; \mathbb{Q})^\vee$ OR DUALLY, $H_c^{2g}(\mathcal{M}_g) = H_{4g-6}(\mathcal{M}_g)$

$\|$

$H_{4g-6}(\mathrm{Mod}_g)$

cochain cpx

GC = "Graph complex", defined combinatorially

$$= \prod_{g\geq 3} GC^{(g)}$$

Goal:

$$H^*(GC^{(g)}) \hookrightarrow H_c^*(\mathcal{M}_g)$$

Domain "large" & has explicit classes & interesting structure + injection onto "large" subspace in $H_c^{2g}(\mathcal{M}_g)$ (eg larger than the image of the K-classes)

$$\begin{tikzcd} H^*(GC) \arrow[r, hook] & \prod_g H_c^*(\mathcal{M}_g) \\ \text{Free Lie}\{\sigma_3, \sigma_5, \ldots\} \arrow[u, hook] & \prod_g H_c^{2g}(\mathcal{M}_g) \arrow[u, hook] \end{tikzcd}$$

odd $g$: $\sigma_g \mapsto H_c^{2g}(\mathcal{M}_g)$

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Mikala's talk: $G = SL_n$, $\Gamma < G(\mathbb{Z}) = SL_n(\mathbb{Z})$

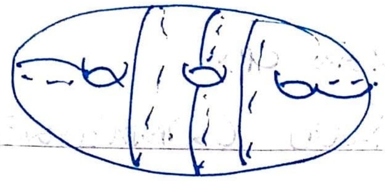
$$\begin{tikzcd} \mathbb{R}^{\ell p}_{>0} \times e_p \subset X \arrow[d, hook] \arrow[loop above, "\Gamma"] = SL_n(\mathbb{R})/SO(n) = G(\mathbb{R})/K \\ \mathbb{R}^{\ell p}_{\geq 0} \times e_p \subset \overline{X} \end{tikzcd}$$

$\mathrm{Mod}_g$

Here: $T_g = \mathrm{Teich}(S_g) \cong \mathbb{R}^{6g-6}$

$$\begin{tikzcd} T_g \arrow[d, hook] \\ \overline{T}_g \end{tikzcd}$$

$\overline{T}_g$ = Harvey bordification

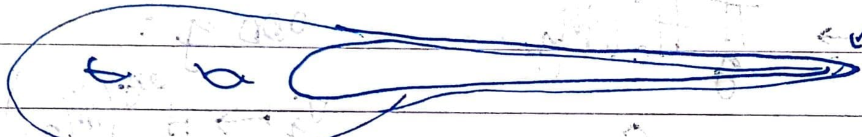
 $=$ $\coprod_{3g-2}$  $/\sim$

↑

3g − 3 CURVES: LENGTH $\mathbb{R}^{3g-3}_{>0}$, TWIST PARAMETER $\mathbb{R}^{3g-3}$

$$\begin{tikzcd}
\mathbb{R}^{3g-3}_{>0} \times \mathbb{R}^{3g-3} \arrow[r, "\cong"] \arrow[d, hook] & \mathcal{T}_g \arrow[d, hook] \\
\mathbb{R}^{3g-3}_{\geq 0} \times \mathbb{R}^{3g-3} \arrow[r] & \hat{\tilde{\mathcal{T}}}_g
\end{tikzcd}$$

(VERY SIMILAR TO BOREL-SERRE)



REMEMBERING THE TWIST PARAMETER

(HENCE ≠ DELIGNE-MUMFORD)

(CAN INSTEAD DO THIS BY DEFINING A CERTAIN SUBSPACE WHERE GEODESICS HAVE LENGTH $\geq \varepsilon$)

COMBINATORICS OF CORNER STRATA & THEIR INCLUSION

$\updownarrow$

$C(S_g)$ IN TEICHMULLER SPACE (OR $\tilde{\mathcal{T}}_g$)

NOT COMPACT BECAUSE OF THE TWIST FACTORS!

IN $\hat{\mathcal{M}}_g = \hat{\mathcal{T}}_g / \mathrm{Mod}_g$

CORNER STRATA

$\updownarrow$

$|\mathcal{C}(S_g)| / \mathrm{Mod}_g$

CAN BE PHRASED IN TERMS OF STABLE GRAPHS INSTEAD:

$(\Gamma, w)$ $\Gamma$ CONNECTED ABSTRACT GRAPH

$w : V \longrightarrow \mathbb{N}$

$$b_1(\Gamma) + \sum_{v \in V(\Gamma)} w(v) = g$$

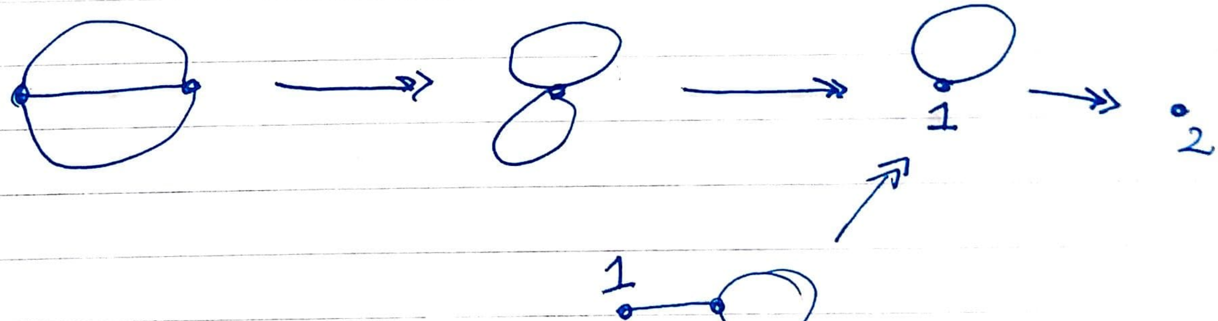
eg. 

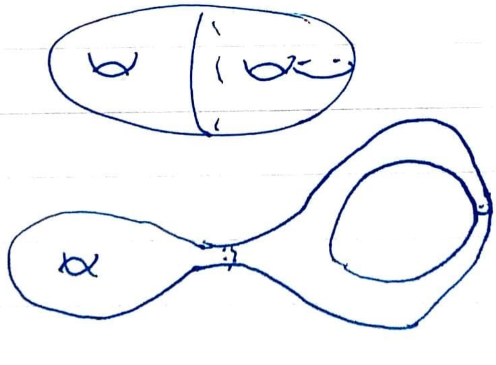

STABLE GRAPH OF GENUS 4.

"STABLE" $\forall v \in V(\Gamma)$,

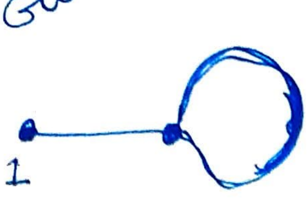
$$|v| + 2w(v) \geq 3.$$

MORPHISMS IN $\mathcal{J}_g$ = CAT OF STABLE GENUS $g$ GRAPHS




VERTEX = COMPONENT
EDGE $\leftrightarrow$ GLUING

$\longmapsto$ 

$\mathcal{C}(S_g)/\mathrm{Mod}_g \longleftrightarrow \mathcal{J}_g^\circ = \mathcal{J}_g \setminus \{\bullet_g\}$.

CURVES $\longleftrightarrow$ EDGES

PATH COMP. OF $S_g \setminus$ CURVE SYSTEM $\longleftrightarrow$ VERTICES

INCLUSION OF SIMPLICES $\longleftrightarrow$ EDGE CONTRACT

NOTE: THERE IS A TERMINAL OBJECT $\bullet_g \in \mathcal{J}_g$

EMPTY CURVE SYSTEM $\longleftrightarrow$ $\bullet_g$

NOTE: NEITHER SIDE IS A POSET. (OR SIMPLICIAL CPX)

NOTE: ALSO EXISTS IN A VERSION WITH MARKED POINTS

$\mathcal{J}_{g,n}$ HAS ALSO $m: \{1, .., n\} \to V(\Gamma)$.