Twisted Homology Operations

Classically:

$X \in \mathrm{Alg}_{E_n}(\mathrm{Top})$. Have operations

Dyer Lashof ops
- $Q_{i(p-1)}: H_q(X;\mathbb{F}_p) \longrightarrow H_{pq+i(p-1)}(X;\mathbb{F}_p)$

  $\forall\ 0 \le i \le n-1$
- $\beta Q_{i(p-1)}: H_q \longrightarrow H_{pq+i(p-1)-1}$ $(p>2)$

  $\forall\ 0 \le i \le n-1$

Browder bracket
- $[\ ,\ ]: H_{q_1} \otimes H_{q_2} \longrightarrow H_{q_1+q_2+n-1}$ $(n<\infty)$
- product

generating all ops.

Cohen, May gave description of H_* of free E_n alg in terms of ops.

Twisted coefficients

Goal: Understand homology of free E_n algs with twisted coefficients

Consider coeff systems
- valued in field $\mathbb{F}$
- pulled back from grp completion

Recall: $X \in \mathrm{Alg}_{E_n}(\mathrm{Top})$, have group completion $X \xrightarrow{E_n} Y \leftarrow$ n-fold loop space

These are det'd by homomorphism
$\varphi: \pi_1 Y \longrightarrow \mathbb{F}^\times$.

Denote system by $\mathbb{F}_\varphi$.

In an appropriate category, can define ops as in classical case.

For $n>2$, get almost same ops as classically.

$\boxed{n=2}$

Twisted Dyer-Lashof ops: come from H_* of braid grps with twisted coeffs.

Computed by Ellenberg-Tran-Westerland.

Story is similar to classical case.

Twisted Browder bracket:

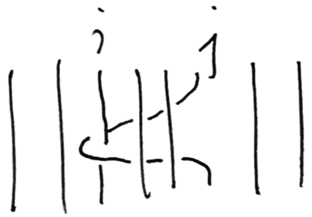
Look for m-ary ops refers to component of X ($a_i \in \pi_0 X$)

$$H_{a_1,n_1}(X;\mathbb{F}_\varphi)\otimes\cdots\otimes H_{a_m,n_m}(X;\mathbb{F}_\varphi)\longrightarrow H_{a_1+\dots+a_m,\,n_1+\dots+n_m+m-1}$$

These ops come from

$$H_{m-1}(Br_m;\bigoplus\mathbb{F})\cong H_{m-1}(PBr_m;\mathbb{F})$$

PBr_m gen'd by $\alpha_{ij}=$



$1\le i<j\le m$

$\alpha_{ij}\circlearrowright\mathbb{F}$ via mult. by $q_{ij}\in\mathbb{F}^\times$. q_{ij} det'd by a_i, a_j, φ, c ← function coming from E_2 str on Y

Computations of Cohen, Arnold $\Rightarrow$ if all $q_{ij}=1$, then $\dim_{\mathbb{F}} H_{m-1}(PBr_m;\mathbb{F})=(m-1)!$

These come from iterated Browder bracket.

Thm: (B.)

(1) $\prod_{1\le i<j\le m} q_{ij}\neq 1 \iff H_{m-1}(PBr_m;\mathbb{F})=0$

(2) If $\prod_{1\le i<j\le m} q_{ij}=1$, and $\prod_{\{i<j\}\subset I} q_{ij}\neq 1 \;\forall I\subsetneq\{1,\dots,m\}$,

then $\dim_{\mathbb{F}} H_{m-1}(PBr_m;\mathbb{F})=(m-2)!$

So in case (2), get $(m-2)!$ lin indep. ops that don't decompose into ops of lower arity – don't have usual Browder bracket, but do have "m-ary bracket"!