Euler class in power subgroup

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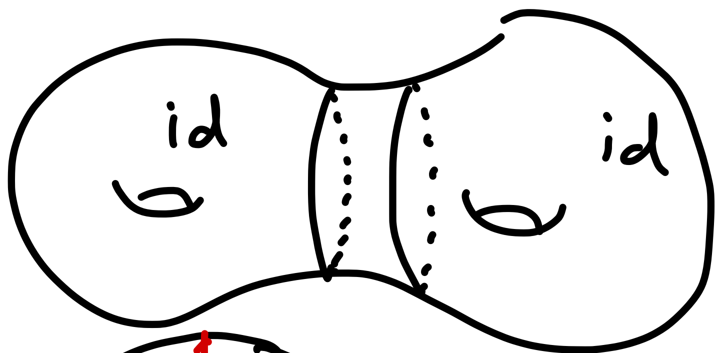
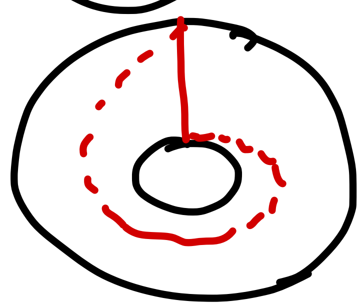
$MCG(S_g) = \pi_0(\mathrm{Homeo}(S_g))$

One phenomenon: $MCG(S_g)$ is generated by **Dehn twist** !





$S^1 \times I$

(θ, t)

$(\theta, t) \xrightarrow{DT} (\theta + t, t)$

$MCG(S_g)$ can be generated by elements that's supported on smallest possible subsurfaces.

What about finite index subgp of $MCG(S_g)$?

Is it generated by elements with small supports?

What about by powers of Dehn twists?

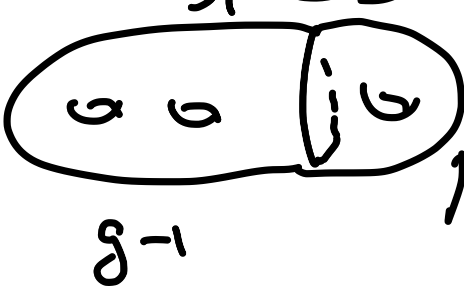
<u>Funar</u>: No! The subgp $P(n)$

$$P(n) = \{ T_d^n \mid \forall d \} \trianglelefteq MCG(S_g)$$

has infinite index.

More generally, what about genus g−1 subsurfaces?

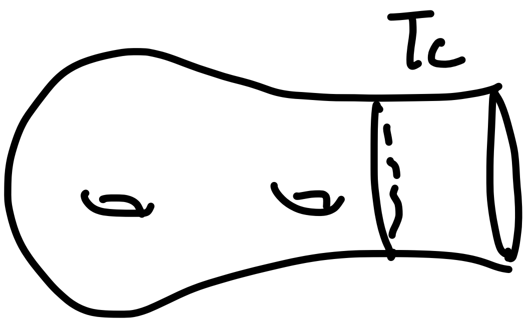
$S_1 \subseteq S$ $\qquad \Gamma \trianglelefteq MCG(S_g)$

Q1: 

$$N = N(\Gamma \cap MCG(S_1)) \trianglelefteq \Gamma$$

Is N finite index in Γ ?

Last year, I tried to study a dynamics problem called Nielsen realization for finite index subgp of <u>MCG</u>.



$T_c \neq 0$

$$\mathrm{Mod}(S_g^1) = \pi_0(\mathrm{Homeo}(S_g^1))$$

$\uparrow$ boundary component.

$\uparrow$ fixing bd pointwise

<u>Statement</u>: T_c or a power of T_c is a product of Dehn twists on nonseparating curves.

For finite index subgp of $MCG(S_g^2)$, is T_c^K a product of Dehn twists on nonseparating curves?

Q2: Is T_c^K a product of classes supported on $g-1$ subsurfaces?

Connect with Euler class

$$1 \to \mathbb{Z} \to G \xrightarrow{\pi} \bar{G} \to 1$$

$$\mathrm{eu}(\pi) \in H^2(\bar{G};\mathbb{Z})$$

$$H_2(\bar{G};\mathbb{Z}) \longrightarrow \mathbb{Z}$$

$f: \pi_1(S_g) \to \bar{G}$

$1 = [a_1 b_1] \cdots [a_g b_g] \rightsquigarrow$

$a_i\ b_i \in \bar{G}$

$\tilde{a}_1\ \tilde{b}_1 \cdots \tilde{a}_g \tilde{b}_g$ lifts to G

$\mathrm{eu}(f) = [\tilde{a}_1, \tilde{b}_1] \cdots [\tilde{a}_g, \tilde{b}_g]$

Cor: $\mathrm{eu}(\pi) = 0 \iff \pi$ has a section.

MCG

one marked point

$$1 \to \mathbb{Z} \xrightarrow{\langle T_c \rangle} MCG(S_g^1) \xrightarrow{\tilde{\pi}} MCG(S_{g,1}) \to 1$$

Fact: $g \geq 2$, this central extension is not a product

$eu \neq 0$

$\Rightarrow T_c^k = [a_1 b_1] \cdots [a_n b_n] \quad a_i \in MCG(S_g^1)$

we can write a_i as product of Dehn twists of nonseparating curves.

$\tilde{\pi}$ has a natural "section"

$$T_a \in MCG(S_{g,1}) \longmapsto T_a \in MCG(S_g^1)$$

This is not a section.

some relation $T_{a_1} \cdots T_{a_k} = 1 \rightsquigarrow T_{a_1} \cdots T_{a_k} = T_c^k$

Fix N_0, we ask

$$T_{a_1}^{N_0} \cdots T_{a_n}^{N_0} = T_c^{k}$$ possible for a nonzero k?

$\Updownarrow$

Equivalently, this is the same as whether $eu \in H^2(P(N_0); \mathbb{Z})$ is trivial or not!

$\Updownarrow$

The natural "section" is an actual section.

<u>Dahmani's Thm</u>

$$MCG(S_g) \quad MCG(S_{g,1}) \rhd P(N)$$

$\exists$ an N_0 s.t N any multiple of N_0 we have $P(N)$ only has two kinds of relations.

① Commutating relation $i(\alpha\beta)=0$

$\Rightarrow T_\alpha^N, T_\beta^N$ commute

② conjugation relation

$$T_\alpha^N T_\beta^N T_\alpha^{-N} = T_{T_\alpha^N(\beta)}$$

This is very not true for $MCG(S)$.

This theorem provides us with the section s that we want.

The "natural section" is an actual section.

$\Downarrow$

$en(P(N))$ is trivial.