Stability in Topology, Arithmetic, and Representation Theory

2022

Abstracts

Saturday, March 26, 2022

9:00 – 9:40: Andrew Snowden

Title: Oligomorphic groups and tensor categories.

Abstract: Deligne constructed an abstract tensor category that, in a certain sense, interpolates the representation categories of finite symmetric groups. We introduce a theory of integration on the infinite symmetric group, and use it to make Deligne's category more concrete: we define a variant of the group algebra of the infinite symmetric group, using convolution of functions, and realize Deligne's category as modules over this algebra. Our approach applies in much greater generality: namely, whenever one has an oligomorphic group and a certain kind of measure on it. This includes all the Deligne categories interpolating families of finite groups, but also leads to new examples where there are no finite groups to interpolate. For instance, we construct a Deligne-like category associated to the homeomorphism group of the real line. Our theory also potentially applies to the automorphism group of the Rado graph; however, in this case it is not clear if an appropriate measure exists.

10:00 – 10:20: Hannah Alpert

Title: Concatenation product on homology of disks in a strip.

Abstract: Inside the configuration space of n points in the plane, we consider the subspace where at most w points may have the same x-coordinate. This is the configuration space of n disks in the strip of width w. There is a multiplication on homology: given a j_1-cycle on n_1 disks and a j_2-cycle on n_2 disks, we can put them side by side to get a $(j_1 + j_2)$-cycle on $(n_1 + n_2)$ disks. For any width w, this makes the homology into a noncommutative algebra when the disks are unordered, and into a twisted noncommutative algebra when the disks are ordered. Is there a finite set of homology classes that generates the whole algebra? It seems so, but is there a way to prove it without computing all of the homology explicitly?

10:30 – 10:50: Mark Shusterman

Title: Finding G-extensions of a function field with minimal ramification

Abstract: Boston and Markin conjectured that every finite group G can be realized as the Galois group of an extension of the rational numbers whose number of ramified primes is the number of generators of the abelianization of G. We explain how a function field (over a large finite field) version of this conjecture is related to a representation stability problem for the symmetric group.

11:00 – 11:20: Sophie Kriz

Title: On representation stability of symmetric groups in positive characteristic.
Abstract: Representation stability of symmetric groups is encoded in the category of generic FI-modules. In characteristic 0, this category, and in particular its simple objects, are well understood by the results of S. Sam and A. Snowden. In this talk, I will discuss my results on the structure of simple generic FI-modules in characteristic $p > 0$. If there is time left, I may also discuss some examples of local cohomology of integral FI-modules.

11:30 – 11:50: Zachary Himes

Title: On (not) the rational dualizing module for $\mathrm{Aut}(F_n)$.
Abstract: Bestvina–Feighn proved that $\mathrm{Aut}(F_n)$ is a rational duality group, i.e. there is a $\mathbb{Q}]\,\mathrm{Aut}(F_n)]$-module, called the rational dualizing module, and a form of Poincare duality relating the rational cohomology of $\mathrm{Aut}(F_n)$ to its homology with coefficients in this module. Bestvina–Feighn's proof does not give an explicit combinatorial description of the rational dualizing module of $\mathrm{Aut}(F_n)$. But, inspired by Borel–Serre's description of the rational dualizing module of arithmetic groups, Hatcher–Vogtmann constructed an analogous module for $\mathrm{Aut}(F_n)$ and asked if it is the rational dualizing module. In work with Miller, Nariman, and Putman, we show that Hatcher–Vogtmann's module is not the rational dualizing module.

2:00 – 2:20: Bena Tshishiku

Title: Counting flat cycles in the homology of locally symmetric spaces
Abstract: We give a lower bound on the growth of the n-th Betti number of congruence subgroups of $\mathrm{SL}(n+1,\mathbb{Z})$. The source of this homology is flat cycles in the associated locally symmetric space. Our techniques can also be applied to other arithmetic groups. This is joint work in progress with Daniel Studenmund.

2:30 – 2:50: Christin Bibby

Title: Homology representations of compactified configurations on graphs applied to moduli spaces of curves
Abstract: We obtain new calculations of the top weight rational cohomology of the moduli space of genus 2 algebraic curves with n marked points, equivalently the rational homology of the moduli space of genus 2 tropical curves with n marked points, as a representation of the symmetric group. These calculations are achieved fully for all $n \leq 10$, and partially (for specific irreducible representations) for $n \leq 22$. We achieve our calculations via a comparison with the homology of compactified configuration spaces of graphs. These homology groups are equipped with commuting actions of a symmetric group and the outer automorphism group of a free group. We construct an efficient free resolution for these homology representations. Using the Peter-Weyl Theorem for symmetric groups, we consider irreducible representations individually, vastly simplifying the calculation of these homology representations from the free resolution. This is joint work with Melody Chan, Nir Gadish, and Claudia He Yun.

3:30 – 3:50: Aida Maraj

Title: Infinitely dimensional algebras, shift operators, and Segre languages.
Abstract: The talk will be about algebras and ideals in polynomial rings in infinitely many variables that are invariant under a shift on the indices of the variables. We analyze when such algebraic objects are finitely generated up to this operator. Then we use regular languages and finite automata to determine the rationality of an equivariant Hilbert series for them. Lastly, we define Segre languages which fit the definition of a Segre product from algebraic geometry.

4:00 – 4:20: Nicholas Wawrykow

Title: Decomposing the rational homology groups of the ordered configuration space of n hard-disks on an infinite strip of width 2 into induced S_n-representations.
Abstract: Alpert proves that the k^{th}-integral homology of the ordered configuration space of n open unit-diameter disks on the infinite strip of width 2 is an FI_{k+1}-module by studying certain operations on homology called "high-insertion maps." The integral homology groups $H_k(\text{cell}(n,2))$ are free abelian, and Alpert computes a basis for $H_k(\text{cell}(n,2))$ as an abelian group. We decompose the rational homology groups of $\text{cell}(n,2)$ into a direct sum of induced representations of the symmetric group S_n. We do this by proving the existence of a basis for $H_1(\text{cell}(n,2);\mathbb{Q})$ that has "nice" representation theoretic properties. We use results of Alpert to help build a basis for $H_k(\text{cell}(n,2);\mathbb{Q})$. This allows us to apply results of Ramos to get a decomposition of $H_1(\text{cell}(n,2);\mathbb{Q})$ into a direct sum of induced S_n-representations.

4:30 – 5:10: Jennifer Wilson

Title: Rognes' connectivity conjecture
Abstract: Rognes' connectivity conjecture concerns the connectivity of a simplicial complex called the common basis complex, with connections to algebraic K-theory. I will describe joint work-in-progress with Miller and Patzt proving the connectivity conjecture for fields.

Sunday, March 27, 2022

9:00 – 9:20: Aaron Landesman

Title: Geometric local systems on very general curves
Abstract: This is joint work with Daniel Litt. We prove a stability result for the absence of polarized variations of Hodge structure with infinite monodromy on very general curves of large genus. As a sample application, fix a relative dimension r. Then, for g more than r^2, any r-dimensional abelian scheme over a very general curve of genus g is isotrivial.

9:30 – 10:10: Daniel Litt

Title: Hodge theory of mapping class group dynamics

Abstract: This is joint work with Aaron Landesman. There are a number of difficult open questions around mapping class group representations and dynamics, which it turns out are accessible to methods from Hodge theory and arithmetic geometry. For example, I'll discuss applications of these methods to the following concrete theorem about surface groups, whose proof relies on non-abelian Hodge theory and the Langlands program:

Theorem. Let $\rho\colon \pi_1(\Sigma_g) \to \mathrm{GL}_r(\mathbb{C})$ be a representation of the fundamental group of a compact orientable surface of genus g, with $r < \sqrt{g}$. If the conjugacy class of ρ has finite orbit under the mapping class group of Σ_g (equivalently, under the outer automorphism group of $\pi_1(\Sigma_g)$), then ρ has finite image.

This answers a question of Peter Whang. I'll also discuss closely related applications to the Putman–Wieland conjecture on homological representations of mapping class groups.

10:30 – 10:50: Calista Bernard

Title: Twisted homology operations for E_2-algebras

Abstract: In the 70s, Fred Cohen and Peter May gave a description of the mod p homology of a free E_n-algebra in terms of certain homology operations, known as Dyer–Lashof operations, and the Browder bracket. I will discuss a framework to generalize these operations to homology with certain twisted coefficient systems and explain an application of this theory for $n = 2$ to mixed braid groups.

11:00 – 11:20: Phil Tosteson

Title: Homological stability and curves on del-Pezzo surfaces

Abstract: Let X be a projective variety, and C be an algebraic curve. The topological problem of computing the homology of the space of algebraic maps from C to X, is analogous to the arithmetic problem of counting the number of rational points on X. I will talk about joint work with Ronno Das, considering the case where X is a del Pezzo surface.

2:00 – 2:20: Uwe Nagel

Title: Asymptotic Stabilization in Sequences of Ideals and Modules

Abstract: Families of polynomial ideals arising in various contexts often have a rich algebraic structure, reflected, for example, in a large symmetry group. It is natural to expect that the properties of related ideals remain somewhat invariant even when the ideals involve more and more variables. We discuss asymptotic results for invariants such as the Hilbert function and multiplicity as well as graded graded syzygies that confirm this expectation.

3:00 – 3:20: Claudio Gómez-Gonzáles

Title: The moduli space of linear systems on $\mathbb{CP}^1$.

Abstract: This talk is based on joint work with Oishee Banerjee, inspired by relating homological stability in spaces of rational maps to predictions from arithmetic via the Grothendieck–Lefschetz trace formula in étale cohomology. We will introduce this motivating phenomena before studying the topology of $\mathrm{Lin}_n^r(\mathbb{C})$, the moduli space of dimension n and degree r linear systems on $\mathbb{CP}^1$ via the

method of symmetric semisimplicial filtrations. We conclude by connecting this calculation to the homological density of the non-degenerate functions in the space of holomorphic functions $\mathbb{CP}^1 \to \mathbb{CP}^1$.

3:30 – 3:50: Nir Gadish

Title: From configurations on graphs to cohomology of $\mathcal{M}_{2,n}$

Abstract: High dimensional cohomology of the moduli spaces provides a huge and mysterious sequence of representations of the symmetric groups that does not stabilize in any obvious way. One window into these cohomology groups is provided by tropical geometry, relating them to configuration spaces on graphs. We will discuss this connection, the computational approaches it affords, and the sense in which representation stability might be manifest after all.