Stability in Topology, Arithmetic, and Representation Theory 2023

Abstracts

Monday, July 17, 2023

9:00 – 9:45: Luciana Basualdo Bonatto

Title: Talk 1: Scanning – from Configuration Spaces to Cobordism Categories
Abstract: Collections of spaces such as configuration spaces and moduli spaces of surfaces exhibit a stability phenomenon in their homology. When analyzing a sequence of spaces that exhibit such stability, a fundamental question arises: "What does it stabilize to?". Determining the stable homology often proves easier than computing the homologies of the original spaces, thanks to a technique known as scanning. In this series of talks, we will explore the scanning technique through two key examples. Firstly, we will discuss Segal's influential work on configuration spaces of points in Euclidean space, which can be viewed as the original scanning argument. Subsequently, we will explore how this technique has been adapted to more intricate settings, such as the study of moduli spaces of manifolds. In the case of surfaces, this technique played a pivotal role in the proof of the Mumford conjecture by Madsen-Weiss. We will discuss this result and its generalizations by Galatius–Madsen–Tillmann–Weiss and Galatius–Randal-Williams to moduli spaces of higher dimensional manifolds and cobordism categories.

10:00 – 10:45: Alexander Kupers

Title: Talk 1: Mapping class groups
Abstract: We will look at Kreck's determination of the mapping class groups of highly-connected high-dimensional manifolds, and explain how it is an example of the general connection between mapping class groups and arithmetic groups discovered by Sullivan.

11:30 – 12:15: Andrew Putman

Title: Talk 1: Representation stability and homological stability
Abstract: I will discuss the following three things:

i) The homological stability machine
ii) Some basic examples and structures from the theory of representation stability
iii) How to incorporate ii) into the argument from i).

2:15 – 3:00: Oishee Banerjee

Title: Stable cohomology and arithmetic of some mapping spaces

Abstract: How do we describe the topology of the space of all nonconstant holomorphic (respectively, algebraic) maps $F : X \to Y$ from one complex manifold (respectively, variety) to another? What is, for example, its cohomology? Such problems are old but difficult, and are nontrivial even when the domain and range are Riemann spheres. In this talk I will explain how these problems relate to other parts of mathematics such as spaces of polynomials, arithmetic (e.g the geometric Manin conjectures), algebraic geometry (e.g. moduli spaces of elliptic fibrations, of smooth sections of a line bundle, etc) and homotopy theory (e.g. derived indecomposables of modules over monoids). I will show how one can fruitfully attack such problems by incorporating techniques from topology to the holomorphic/algebraic world (e.g. by constructing a new spectral sequence).

3:15 – 4:00: Anh Trong Nam Hoang

Title: Configuration spaces and applications in arithmetic statistics

Abstract: In the last dozen years, topological methods have been shown to produce a new pathway to study arithmetic statistics over function fields, most notably in Ellenberg–Venkatesh–Westerland's work on the Cohen–Lenstra conjecture. More recently, Ellenberg, Tran and Westerland proved the upper bound in Malle's conjecture over function fields by studying stability of the homology of braid groups with certain exponential coefficients. In this talk, we will give an overview of their framework and extend their techniques to study other questions in arithmetic statistics. As an example, we will demonstrate how this extension can be used to study character sums of the resultant of monic square-free polynomials over finite fields, answering and generalizing a question of Ellenberg and Shusterman.

4:15 – 5:00: Nir Gadish

Title: A Serre spectral sequence for moduli spaces of tropical curves.

Abstract: The moduli space of genus g tropical curves with n marked points is a fascinating topological space, with a combinatorial flavor and deep algebro-geometric meaning. In the algebraic world, forgetting the n marked points gives a fibration whose fibers are configuration spaces of a surface, and Serre's spectral sequence lets one compute the cohomology "in principle". In joint work with Bibby, Chan and Yun, we construct a surprising tropical analog of this spectral sequence, manifesting as a graph complex of and featuring the cohomology of compactified configuration spaces on graphs.

Tuesday, July 18, 2023

9:00 – 9:45: Luciana Basualdo Bonatto

Title: Talk 2: Scanning – from Configuration Spaces to Cobordism Categories

Abstract: See above

10:00 – 10:45: Alexander Kupers

Title: Talk 2: Diffeomorphisms

Abstract: Galatius and Randal-Williams proved that the homological stability and stable homology results for mapping class groups of surfaces have analogues in high dimensions. We will explain some of their techniques, focusing on the connection to arithmetic groups discussed in the previous lecture.

11:30 – 12:15: Andrew Putman

Title: Talk 2: Representation stability and homological stability

Abstract: See above

2:15 – 3:00: Sophie Kriz

Title: T-algebras and the Vector Delannoy Category

Abstract: I will talk about a universal algebra technique for constructing additive categories with ACU tensor product and strong duality. As an application, I will discuss a new semisimple pre-Tannakian category which is a "$q > 1$"-version of the Delannoy category of Harman, Snowden, and Snyder. Using a recent result of A. Snowden, I obtain examples of semisimple pre-Tannakian categories of growth $\exp(\exp(cn^2))$, the highest currently known.

3:15 – 3:30: Connor Malin

Title: Differentiating automorphisms of the E_n-operad

Abstract: It has been known since Wall and Spivak initiated the study of Poincaré complexes that it is possible to differentiate arbitrary homotopy equivalences of Poincaré complexes using the Spivak normal fibration. We describe a lift of this construction to operads and compute its value on $O(n) \leq \mathrm{hAut}(E_n)$.

3:45 – 4:00: Mathieu de Langis

Title: Towards a Topological Proof of Wright's Theorem

Abstract: Malle's Conjecture concerns the asymptotic behavior of the number of degree n extensions of a number field with Galois group permutation-isomorphic to G. Using the function field analogy, a similar conjecture can be made for finite extensions of $\mathbb{F}_q(t)$, where q is a power of a prime. In the case where G is abelian, Wright has proven a version of Malle's conjecture in both contexts, using class field theory.

This talk concerns an alternative approach towards proving Malle's conjecture in the function field case, by translating the problem into one of counting points on Hurwitz spaces.

4:15 – 5:00: Christin Bibby

Title: Supersolvable posets and fiber-type abelian arrangements

Abstract: We present a combinatorial analysis of fiber bundles of generalized configuration spaces on connected abelian Lie groups. These bundles are akin to those of Fadell-Neuwirth for configuration spaces, and their existence is detected by a combinatorial property of an associated finite partially ordered set. This is consistent with Terao's fibration theorem connecting bundles of hyperplane arrangements to Stanley's lattice supersolvability. We obtain a combinatorially determined class of $K(\pi, 1)$ toric and elliptic arrangements. Under a stronger combinatorial condition, we prove a factorization of the Poincaré polynomial when the Lie group is noncompact. In the case of toric arrangements, this provides an analogue of Falk–Randell's formula relating the Poincaré polynomial to the lower central series of the fundamental group. This is joint work with Emanuele Delucchi.

Wednesday, July 19, 2023

9:00 – 9:45: Luciana Basualdo Bonatto

Title: Talk 3: Scanning – from Configuration Spaces to Cobordism Categories
Abstract: See above

10:00 – 10:45: Alexander Kupers

Title: Talk 3: Self-embeddings
Abstract: One way in which high-dimensional manifolds differ from low-dimensional ones, is that there is a separate tool to study their self-embeddings: the embedding calculus of Goodwillie, Klein, and Weiss. We will explain this tool and outline some applications related to the previous two lectures.

11:30 – 12:15: Andrew Putman

Title: Talk 3: Representation stability and homological stability
Abstract: See above

2:15 – 2:30: Zachery Himes

Title: A lower bound on the top degree rational cohomology of the symplectic group of a number ring
Abstract: Let R be a number ring. If one fixes i and lets n go to infinity, then the rational cohomology $H^i(\mathrm{SL}_n(R); \mathbb{Q})$ stabilizes in a range. Outside this range, little is known about the rational cohomology in general except that it vanishes for all $i > \nu_n$, where ν_n is an explicit constant calculated by Borel–Serre. For $i = \nu_n$, Church–Farb–Putman recently showed that the dimension of $H^{\nu_n}(\mathrm{SL}_n(R); \mathbb{Q})$ is at least $(|\mathrm{Cl}(R)| - 1)^{n-1}$, where $\mathrm{Cl}(R)$ denotes the class group of R. For the rational cohomology of the symplectic group $\mathrm{Sp}_{2n}(R)$, similar stability and vanishing patterns occur. In joint work with Benjamin Brück, we obtain a similar lower bound for the the top degree rational cohomology of $\mathrm{Sp}_{2n}(R)$ and show it has dimension at least $(|\mathrm{Cl}(R)| - 1)^n$.

2:45 – 3:00: Matthew Scalamandre

Title: A Solomon-Tits theorem for $\mathrm{GL}_n(\mathbb{Z}/N)$.
Abstract: The classical Solomon–Tits theorem states that a spherical Tits building over a field is homotopy equivalent to a wedge of spheres of the appropriate dimension. In this talk, we'll define a Tits complex that makes sense for an arbitrary ring, and prove a Solomon-Tits theorem for most Artin

rings (in particular, for $\mathbb{Z}/N$). We will discuss applications to the cohomology of principal congruence subgroups of $\mathrm{SL}_n(\mathbb{Z})$, and some results about the top homology of this complex (an analogue of the classical Steinberg module).

3:15 – 4:00: Daniel Minahan

Title: Homological Finiteness in the Torelli group.

Abstract: The Torelli group of a closed surface of genus g is the kernel of the symplectic representation of the mapping class group. We will show that for all sufficiently large genera g, the second rational homology of the Torelli group of a closed surface of genus g is finite dimensional.

Thursday, July 20, 2023

9:00 – 9:45: Andrea Bianchi

Title: Homology of configuration spaces of surfaces modulo an odd prime

Abstract: This is joint work with Andreas Stavrou. For a compact orientable surface S of genus g with one boundary component and for an odd prime number p, we study the homology of the unordered configuration spaces $C(S) := \coprod_{n\geq 0} C_n(S)$ with coefficients in $\mathbb{F}_p$. We describe $H_*(C(S);\mathbb{F}_p)$ as a bigraded module over the Pontryagin ring $H_*(C(D);\mathbb{F}_p)$, where D is a disc, and give a splitting as direct sum of certain well-behaved quotients of this ring. We also consider the action of the mapping class group $\mathrm{Mod}(S)$ on the homology, and identify the kernel of the action with the subgroup of $\mathrm{Mod}(S)$ generated by separating Dehn twists and p-th powers of Dehn twists.

10:00 – 10:45: Csaba Nagy

Title: Groups of 8-manifolds

Abstract: We introduce a group $\Theta_8(r)$ consisting of polarised simply-connected 8-manifolds whose homology is isomorphic to that of the connected sum of r copies of $S^2 \times S^6$. The group $\Theta_8(r)$ comes equipped with an action of $\mathrm{GL}(r,\mathbb{Z})$ and a stabilisation map to $\Theta_8(r+1)$. We determine the isomorphism type of $\Theta_8(r)$, and show that the rationalisation of $\Theta_8(r)$, with the action of $\mathrm{GL}(r,\mathbb{Q})$, satisfies representation stability.

11:30 – 12:15: Ismael Sierra

Title: Homological stability of quadratic symplectic groups

Abstract: Quadratic symplectic groups are automorphism groups of non-degenerate, skew-symmetric forms with quadratic refinements. I will talk about improvements in their homological stability, and explain the main steps of the proof, which is based on the cellular E_k-algebras method. Finally, I will comment on the connection between this family of groups and diffeomorphism groups of high dimensional manifolds, and I will state some conjectures for both quadratic symplectic groups and diffeomorphism groups.

2:15 – 3:00: Robin Sroka

Title: Simplicial bounded cohomology and stability
Abstract: Bounded cohomology is a "norm-enriched" version of classical group cohomology, which has applications in geometry and geometric topology. Monod and, recently, De la Cruz Mengual–Hartnick introduced homological stability ideas to the field. In analogy with Quillen's classical approach, the key input for proving stability in bounded cohomology is a bounded acyclicity theorem for certain semi-simplicial sets. In this talk, I will outline an approach for studying such acyclicity properties using "norm-enriched" refinements of well-known simplicial techniques and discuss applications in the realm of stability. This talk is based on joint work with Thorben Kastenholz.

3:15 – 4:00: Adela Zhang

Title: Operations on mod p TAQ cohomology and spectral partition Lie algebras.
Abstract: The bar spectral sequence for algebras over a spectral operad relates Koszul duality phenomena in several contexts. We apply this classical tool to the Koszul dual pair given by the (non-unital) E_∞ operad. The bar spectral sequence for an E_∞ algebras A (over $H\mathbb{F}_p$) computes the derived indecomposables of A, which heuristically counts the number of E_∞ cells one needs to recover A. When applied to the trivial E_∞-$H\mathbb{F}_p$ algebras, we obtain the structure of operations on mod p TAQ cohomology and spectral partition Lie algebras, building on the work of Brantner–Mathew. In the colimit, the unary operations are Koszul dual to the Dyer–Lashof algebra. T is also a shifted restricted Lie structure that can be detected by the homotopy fixed points spectral sequence.

Friday, July 21, 2023

9:00 – 9:15: Evgeniya Lagoda

Title: k-regular maps and the unordered configuration spaces
Abstract: A map from a space X into the space $\mathbb{R}^n$ or $\mathbb{C}^n$ is called *k-regular* if it maps any k distinct points to k linearly independent vectors. One approach to say something about the (non)existence of such maps is to compute characteristic classes of the inverse of the permutation representation bundle over the space of k distinct (unlabeled) points in X. I will explain this approach, apply it to the case $X = \mathbb{R}^d$, and use the classical theory of homology operations, developed by F.R. Cohen, in the assessment of the goodness of our computations.

9:40 – 10:00: Karthik Ganapathy

Title: The infinite variable polynomial ring in positive characteristic
Abstract: GL-equivariant modules over infinite-variable polynomial rings have found applications in topology, algebraic statistics and combinatorics. The structure theory of such modules is well understood in characteristic zero by the work of Sam–Snowden and others. I will talk about ongoing work to extend their results to positive characteristic emphasizing the key differences.

10:15 – 11:00: Nicholas Wawrykow

Title: Representation Stability and Disk Configuration Spaces

Abstract: Church–Ellenberg–Farb and Miller–Wilson proved that for a nice enough manifold X and fixed integer k, the k-th homology group of the ordered configuration space of points in X stabilizes in a representation-theoretic sense as the number of points in the configuration space increases. By fixing a metric on X and replacing points with open unit-diameter disks, we get a new family of configuration spaces where the geometry of X comes to the forefront. One of the simplest of these disk configuration spaces is $\mathrm{Conf}(n, w)$, the ordered configuration space of unit-diameter disks in the infinite strip of width w. The homology groups of $\mathrm{Conf}(*, w)$ do not stabilize in the sense of Church–Ellenberg–Farb, Miller–Wilson; however, Alpert proved that when the width is 2 they stabilize in a related sense. Alpert's methods do not extend to larger widths. In this talk I discuss various notions of representation stability, and show that when w is at least 2 there is a sense in which the rational homology groups of $\mathrm{Conf}(*, w)$ stabilize.

11:00 – 11:45: Philip Tosteson

Title: The category of finite sets and lie algebras of vector fields

Abstract: Representations of the category of finite sets (and its opposite) have been used to establish stability phenomena for certain compactifications of hyperplane arrangements and moduli spaces of curves. However, unlike the case of FI-modules, we do not know a structure theory for these representations. I will talk about joint work with Sam and Snowden which connects these representations to representations of lie algebras of polynomial vector fields (known as Witt algebras). This lets us interpret certain features of this representations geometrically (e.g. a chain complex constructed by Wiltshire-Gordon is related to the de Rham complex).