1. Polynomial $N(L, R_0, R_1)$

```
In[•]:= Clear[denomi0, polyN, equaN];
denomi0[L_, R_, S_] := 512 R^2 S^2 Sqrt[L (R - L) (S - L) (R + S - L)] (2 (R - L) (S - L) - R * S);
polyN[L_, R_, S_, b02_, b12_, b13_] := (16 - 2 b02) R S L^4 + (176 - 2 b13) R^2 S^2 L^2 +
    14 R^3 S^3 + b02 (R^2 + S^2) L^4 + (-48 - b12) (R^3 + S^3) L^3 + b12 (R^2 S + S^2 R) L^3 + 8 (R^4 + S^4) L^2 +
    b13 (R^3 S + S^3 R) L^2 - 16 (R^4 S + S^4 R) L - 56 (R^3 S^2 + S^3 R^2) L + 7 (R^4 S^2 + S^4 R^2);
equaN[R_, S_, b02_, b12_, b13_] := 3 polyN[1, R, S, b02, b12, b13] -
    denomi0[1, R, S] * twist2[{1, R, 0, 0, S, 0, 0}];

In[•]:= Solve[equaN[1.1, 1.2, b02, b12, b13] == 0 && equaN[1.1, 1.3, b02, b12, b13] == 0 &&
  equaN[1.2, 1.3, b02, b12, b13] == 0, {b02, b12, b13}]

Out[•]= {{b02 → 8., b12 → -32., b13 → 48.}}

In[•]:= Clear[finalN, tw2Ord0];
finalN[L_, R_, S_] := polyN[L, R, S, 8, -32, 48];
tw2Ord0[L_, R_, S_] := 3 finalN[L, R, S] / denomi0[L, R, S];

In[•]:= Simplify[finalN[L, R, S]]

Out[•]= 7 R^2 S^2 (R + S)^2 + 8 L^4 (R^2 + S^2) + 8 L^2 (R + S)^2 (R^2 + 4 R S + S^2) -
  16 L^3 (R^3 + 2 R^2 S + 2 R S^2 + S^3) - 8 L R S (2 R^3 + 7 R^2 S + 7 R S^2 + 2 S^3)
```

2. Polynomial $Q(L, R_0, R_1)$

```
Clear[denomi22, symQ, eqsymQ];
denomi22[L_, R_, S_] :=
  1536 Sqrt[L (R - L) (S - L) (R + S - L)] (R - L)^2 (S - L)^2 (2 (R - L) (S - L) - R S) (R + S - L)^2;
symQ[L_, R_, a0_, a1_, a2_, a3_, a4_, a5_, a6_, a7_, a8_] :=
  a0 L^10 + a1 R L^9 + a2 R^2 L^8 + a3 R^3 L^7 + a4 R^4 L^6 + a5 R^5 L^5 + a6 R^6 L^4 + a7 R^7 L^3 + a8 R^8 L^2;
eqsymQ[R_, a0_, a1_, a2_, a3_, a4_, a5_, a6_, a7_, a8_] :=
  symQ[1, R, a0, a1, a2, a3, a4, a5, a6, a7, a8] -
   denomi22[1, R, R] (twist2[{1, R, 1, 0, R, 1, 0}] + tw2Ord0[1, R, R] -
      twist2[{1, R, 1, 0, R, 0, 0}] - twist2[{1, R, 0, 0, R, 1, 0}]);

In[•]:= Solve[eqsymQ[1.1, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0 &&
  eqsymQ[1.2, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0 &&
  eqsymQ[1.3, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0 &&
  eqsymQ[1.4, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0 &&
  eqsymQ[1.6, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0 &&
  eqsymQ[1.7, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0 &&
  eqsymQ[1.8, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0 &&
  eqsymQ[2.0, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0 &&
  eqsymQ[2.1, a0, a1, a2, a3, a4, a5, a6, a7, a8] == 0, {a0, a1, a2, a3, a4, a5, a6, a7, a8}]

Out[•]= {{a0 → -0.0000906187, a1 → 0.000488264, a2 → -64.0011,
  a3 → 384.002, a4 → -930.001, a5 → 1160., a6 → -780., a7 → 264., a8 → -34.}}

In[•]:= Clear[symQfinal];
symQfinal[L_, R_] := symQ[L, R, 0, 0, -64, 384, -930, 1160, -780, 264, -34];
```

```
In[•]:= Simplify[symQfinal[L, R]]
Out[•]= -2 L^2 (L - R)^4 R^2 (32 L^2 - 64 L R + 17 R^2)

In[•]:= Clear[polyQ, eqQ];
polyQ[L_, R_, S_, c20_, c30_, c40_, c31_, c50_, c41_, c51_, c42_, c52_, c53_] :=
  L^8 (c20 (R^2 - 2 R * S + S^2) - 64 * R * S) +
   L^7 (c30 (R^3 + S^3 - R^2 S - S^2 R) + 192 (R^2 S + S^2 R)) +
   L^6 (c40 (R^4 - 2 R^2 S^2 + S^4) + c31 (R^3 S - 2 R^2 S^2 + S^3 R) - 930 R^2 S^2) +
   L^5 (c50 (R^5 + S^5 - R^3 S^2 - S^3 R^2) +
      c41 (R^4 S + S^4 R - R^3 S^2 - S^3 R^2) + 580 (R^3 S^2 + S^3 R^2)) +
   L^4 (c51 (R^5 S + S^5 R - 2 R^3 S^3) + c42 (R^4 S^2 + S^4 R^2 - 2 R^3 S^3) - 780 R^3 S^3) +
   L^3 (c52 * (R^5 S^2 + S^5 R^2 - R^4 S^3 - S^4 R^3) + 132 (R^4 S^3 + S^4 R^3)) +
   L^2 (c53 * (R^5 S^3 + S^5 R^3 - 2 R^4 S^4) - 34 R^4 S^4);
eqQ[R_, S_, c20_, c30_, c40_, c31_, c50_, c41_, c51_, c42_, c52_, c53_] :=
  polyQ[1, R, S, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] -
   denomi22[1, R, S] (twist2[{1, R, 1, 0, S, 1, 0}] + tw2Ord0[1, R, S] -
      twist2[{1, R, 1, 0, S, 0, 0}] - twist2[{1, R, 0, 0, S, 1, 0}]);

In[•]:= Solve[eqQ[1.02, 1.1, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.02, 1.2, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.02, 1.3, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.04, 1.4, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.04, 1.5, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.04, 1.6, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.07, 1.7, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.07, 1.8, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.09, 1.9, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0 &&
  eqQ[1.09, 2.0, c20, c30, c40, c31, c50, c41, c51, c42, c52, c53] == 0,
 {c20, c30, c40, c31, c50, c41, c51, c42, c52, c53}]
Out[•]= {{c20 → 0.0000198841, c30 → -0.0000596248,
  c40 → 0.0000592012, c31 → -192., c50 → -0.0000194786, c41 → 63.9999,
  c51 → 0.0000564052, c42 → -162., c52 → -0.0000543505, c53 → 0.0000174277}}

In[•]:= Clear[finalQ, eqfinalQ];
finalQ[L_, R_, S_] := polyQ[L, R, S, 0, 0, 0, -192, 0, 64, 0, -162, 0, 0];
eqfinalQ[R_, S_] :=
  finalQ[1, R, S] - denomi22[1, R, S] (twist2[{1, R, 1, 0, S, 1, 0}] + tw2Ord0[1, R, S] -
      twist2[{1, R, 1, 0, S, 0, 0}] - twist2[{1, R, 0, 0, S, 1, 0}]);

In[•]:= Simplify[finalQ[L, R, S]]
Out[•]= -2 L^2 (L - R)^2 R (L - S)^2 S (32 L^2 + 17 R S - 32 L (R + S))
```

3. Polynomial $P(L, R_0, R_1)$

3.1 Find the symmetric part symP of P

```
Clear[symPsU, eqsPsU]; (* use the variables U=
  (R_0+R_1)/2 and V=(R_0-R_1)/2 assuming U is same for each group of data*)
symPsU[U_, V_, ua6_, ua4_, ua2_, ua0_] :=
   (1 - U)^4 (24 - 192 U + 456 U^2 - 384 U^3 + 79 U^4) + ua6 * V^2 + ua4 * V^4 + ua2 * V^6 + ua0 * V^8;
(*symmetric part of P with same variable U*)
eqsPsU[R_, S_, ua6_, ua4_, ua2_, ua0_] := symPsU[(R + S) / 2, (R - S) / 2, ua6, ua4, ua2, ua0] -
    ((twist2[{1, R, 1, 0, S, 1, 0}] + twist2[{1, R, -1, 0, S, -1, 0}] - 2 tw2Ord0[1, R, S])
        denomi22[1, R, S] - 2 finalQ[1, R, S]) / 4;
```

In[ ]:=
```
Clear[dataA1, dataA2, dataA3, dataA4, dataA5, dataA6, dataA7];
dataA1 = Solve[eqsPsU[2.1, 1.9, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.2, 1.8, ua6, ua4, ua2, ua0] == 0 && eqsPsU[2.3, 1.7, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.4, 1.6, ua6, ua4, ua2, ua0] == 0, {ua6, ua4, ua2, ua0}];
dataA2 = Solve[eqsPsU[2.2, 2.0, ua6, ua4, ua2, ua0] == 0 &&
     eqsPsU[2.3, 1.9, ua6, ua4, ua2, ua0] == 0 && eqsPsU[2.4, 1.8, ua6, ua4, ua2, ua0] == 0 &&
     eqsPsU[2.5, 1.7, ua6, ua4, ua2, ua0] == 0, {ua6, ua4, ua2, ua0}];
dataA3 = Solve[eqsPsU[2.3, 2.1, ua6, ua4, ua2, ua0] == 0 &&
     eqsPsU[2.4, 2.0, ua6, ua4, ua2, ua0] == 0 && eqsPsU[2.5, 1.9, ua6, ua4, ua2, ua0] == 0 &&
     eqsPsU[2.6, 1.8, ua6, ua4, ua2, ua0] == 0, {ua6, ua4, ua2, ua0}];
dataA4 = Solve[eqsPsU[2.4, 2.2, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.5, 2.1, ua6, ua4, ua2, ua0] == 0 && eqsPsU[2.6, 2.0, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.7, 1.9, ua6, ua4, ua2, ua0] == 0, {ua6, ua4, ua2, ua0}];
dataA5 = Solve[eqsPsU[2.5, 2.3, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.6, 2.2, ua6, ua4, ua2, ua0] == 0 && eqsPsU[2.7, 2.1, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.8, 2.0, ua6, ua4, ua2, ua0] == 0, {ua6, ua4, ua2, ua0}];
dataA6 = Solve[eqsPsU[2.6, 2.4, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.7, 2.3, ua6, ua4, ua2, ua0] == 0 && eqsPsU[2.8, 2.2, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.9, 2.1, ua6, ua4, ua2, ua0] == 0, {ua6, ua4, ua2, ua0}];
dataA7 = Solve[eqsPsU[2.7, 2.5, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[2.8, 2.4, ua6, ua4, ua2, ua0] == 0 && eqsPsU[2.9, 2.3, ua6, ua4, ua2, ua0] == 0 &&
    eqsPsU[3.0, 2.2, ua6, ua4, ua2, ua0] == 0, {ua6, ua4, ua2, ua0}];
```

In[ ]:=
```
dataA1
```

Out[ ]=
```
{{ua6 → -1136., ua4 → 23., ua2 → 250., ua0 → -17.}}
```

```
(*a0=-17*)
```

```
Clear[fUa2, fUa4, fUa6];
fUa2[U_, ua20_, ua21_, ua22_] := ua20 + ua21 * U + ua22 * U^2;
fUa4[U_, ua40_, ua41_, ua42_, ua43_, ua44_] :=
  ua40 + ua41 * U + ua42 * U^2 + ua43 * U^3 + ua44 * U^4;
fUa6[U_, ua60_, ua61_, ua62_, ua63_, ua64_, ua65_, ua66_] :=
  ua60 + ua61 * U + ua62 * U^2 + ua63 * U^3 + ua64 * U^4 + ua65 * U^5 + ua66 * U^6;
```

```
Clear[x1, x2, x3];
x1 = ua2 /. Part[Part[dataA1, 1], 3];
x2 = ua2 /. Part[Part[dataA2, 1], 3];
x3 = ua2 /. Part[Part[dataA3, 1], 3];
```

In[•]:=
```
Solve[fUa2[2.0, ua20, ua21, ua22] == x1 && fUa2[2.1, ua20, ua21, ua22] == x2 &&
  fUa2[2.2, ua20, ua21, ua22] == x3, {ua20, ua21, ua22}]
```

Out[•]=
```
{{ua20 → 114., ua21 → -372., ua22 → 220.}}
```

```
Clear[y1, y2, y3, y4, y5];
y1 = ua4 /. Part[Part[dataA1, 1], 2];
y2 = ua4 /. Part[Part[dataA2, 1], 2];
y3 = ua4 /. Part[Part[dataA3, 1], 2];
y4 = ua4 /. Part[Part[dataA4, 1], 2];
y5 = ua4 /. Part[Part[dataA5, 1], 2];
```

In[•]:=
```
Solve[fUa4[2.0, ua40, ua41, ua42, ua43, ua44] == y1 &&
  fUa4[2.1, ua40, ua41, ua42, ua43, ua44] == y2 &&
  fUa4[2.2, ua40, ua41, ua42, ua43, ua44] == y3 &&
  fUa4[2.3, ua40, ua41, ua42, ua43, ua44] == y4 &&
  fUa4[2.4, ua40, ua41, ua42, ua43, ua44] == y5, {ua40, ua41, ua42, ua43, ua44}]
```

Out[•]=
```
{{ua40 → -41.0027, ua41 → 548.005, ua42 → -1410., ua43 → 1196., ua44 → -310.}}
```

In[•]:=
```
Clear[z1, z2, z3, z4, z5, z6, z7];
z1 = ua6 /. Part[Part[try1, 1], 1];
z2 = ua6 /. Part[Part[try2, 1], 1];
z3 = ua6 /. Part[Part[try3, 1], 1];
z4 = ua6 /. Part[Part[try4, 1], 1];
z5 = ua6 /. Part[Part[try5, 1], 1];
z6 = ua6 /. Part[Part[try6, 1], 1];
z7 = ua6 /. Part[Part[try7, 1], 1];
```

In[•]:=
```
Solve[fUa6[2.0, ua60, ua61, ua62, ua63, ua64, ua65, ua66] == z1 &&
  fUa6[2.1, ua60, ua61, ua62, ua63, ua64, ua65, ua66] == z2 &&
  fUa6[2.2, ua60, ua61, ua62, ua63, ua64, ua65, ua66] == z3 &&
  fUa6[2.3, ua60, ua61, ua62, ua63, ua64, ua65, ua66] == z4 &&
  fUa6[2.4, ua60, ua61, ua62, ua63, ua64, ua65, ua66] == z5 &&
  fUa6[2.5, ua60, ua61, ua62, ua63, ua64, ua65, ua66] == z6 &&
  fUa6[2.6, ua60, ua61, ua62, ua63, ua64, ua65, ua66] == z7,
 {ua60, ua61, ua62, ua63, ua64, ua65, ua66}]
```

Out[•]=
```
{{ua60 → -215.99, ua61 → 1151.97, ua62 → -2221.97,
  ua63 → 1783.98, ua64 → -401.994, ua65 → -124.001, ua66 → 28.0001}}
```

```
Clear[symPsUnew, eqsPsUnew]; (*doulbe chck: update our function symP*)
symPsUnew[U_, V_, a60_, a61_, a62_, a63_, a64_, a65_, a66_] :=
  (1 - U)^4 (24 - 192 U + 456 U^2 - 384 U^3 + 79 U^4) +
   (a60 + a61 U + a62 U^2 + a63 U^3 + a64 U^4 + a65 U^5 + a66 U^6) * V^2 +
   (-41 + 548 U - 1410 U^2 + 1196 U^3 - 310 U^4) * V^4 + (114 - 372 U + 220 U^2) * V^6 - 17 V^8;
(*symmetric part of P with same variable U*)
eqsPsUnew[R_, S_, a60_, a61_, a62_, a63_, a64_, a65_, a66_] :=
  symPsUnew[(R + S) / 2, (R - S) / 2, a60, a61, a62, a63, a64, a65, a66] -
   ((twist2[{1, R, 1, 0, S, 1, 0}] + twist2[{1, R, -1, 0, S, -1, 0}] - 2 tw2Ord0[1, R, S])
      denomi22[1, R, S] - 2 finalQ[1, R, S]) / 4;
```

In[•]:=
```
Solve[eqsPsUnew[1.03, 1.1, a60, a61, a62, a63, a64, a65, a66] == 0 &&
  eqsPsUnew[1.03, 1.2, a60, a61, a62, a63, a64, a65, a66] == 0 &&
  eqsPsUnew[1.05, 1.3, a60, a61, a62, a63, a64, a65, a66] == 0 &&
  eqsPsUnew[1.05, 1.4, a60, a61, a62, a63, a64, a65, a66] == 0 &&
  eqsPsUnew[1.07, 1.5, a60, a61, a62, a63, a64, a65, a66] == 0 &&
  eqsPsUnew[1.07, 1.6, a60, a61, a62, a63, a64, a65, a66] == 0 &&
  eqsPsUnew[1.09, 1.7, a60, a61, a62, a63, a64, a65, a66] == 0,
 {a60, a61, a62, a63, a64, a65, a66}]
```

Out[•]=
```
{{a60 → -216., a61 → 1152., a62 → -2222., a63 → 1784., a64 → -402., a65 → -124., a66 → 28.}}
```

In[•]:=
```
Clear[symPU, symP];
symPU[U_, V_] := (1 - U)^4 (24 - 192 U + 456 U^2 - 384 U^3 + 79 U^4) +
   (-216 + 1152 U - 2222 U^2 + 1784 U^3 - 402 U^4 - 124 U^5 + 28 U^6) * V^2 +
   (-41 + 548 U - 1410 U^2 + 1196 U^3 - 310 U^4) * V^4 + (114 - 372 U + 220 U^2) * V^6 - 17 V^8;
symP[R_, S_] := symPU[(R + S) / 2, (R - S) / 2];
```

3.2 Find the anti-symmetric part of P

4. The polynomial $S(L, R_0, R_1)$

4.1. The symmetric part of S(L,R,S)

```
Clear[denoS, symSsU, eqsSsU];
denoS[R_, S_] :=
  -768 Sqrt[(R - 1) (S - 1) (R + S - 1)] (R - 1) (S - 1) (R + S - 1) (2 (R - 1) (S - 1) - R * S);
symSsU[U_, V_, sa5_, sa3_, sa1_] :=
  2 (27 U^2 - 40 U + 10) (U - 1)^2 (2 (U - 1)^2 - U^2) + sa5 * V^2 + sa3 * V^4 + sa1 * V^6;
eqsSsU[R_, S_, sa5_, sa3_, sa1_] := symSsU[(R + S) / 2, (R - S) / 2, sa5, sa3, sa1] -
   ((twist2[{1, R, 1, 0, S, 0, 0}] - tw2Ord0[1, R, S] - finalP[R, S] / denomi22[1, R, S]) R +
      (twist2[{1, R, 0, 0, S, 1, 0}] - tw2Ord0[1, R, S] - finalP[S, R] / denomi22[1, R, S])
       S) denoS[R, S] / 2;
```

```
Clear[dataC1, dataC2, dataC3, dataC4, dataC5, dataC6];
dataC1 = Solve[eqsSsU[2.1, 1.9, sa5, sa3, sa1] == 0 && eqsSsU[2.2, 1.8, sa5, sa3, sa1] == 0 &&
    eqsSsU[2.3, 1.7, sa5, sa3, sa1] == 0, {sa5, sa3, sa1}];
dataC2 = Solve[eqsSsU[2.2, 2.0, sa5, sa3, sa1] == 0 && eqsSsU[2.3, 1.9, sa5, sa3, sa1] == 0 &&
     eqsSsU[2.4, 1.8, sa5, sa3, sa1] == 0, {sa5, sa3, sa1}];
dataC3 = Solve[eqsSsU[2.3, 2.1, sa5, sa3, sa1] == 0 && eqsSsU[2.4, 2.0, sa5, sa3, sa1] == 0 &&
     eqsSsU[2.5, 1.9, sa5, sa3, sa1] == 0, {sa5, sa3, sa1}];
dataC4 = Solve[eqsSsU[2.4, 2.2, sa5, sa3, sa1] == 0 && eqsSsU[2.5, 2.1, sa5, sa3, sa1] == 0 &&
     eqsSsU[2.6, 2.0, sa5, sa3, sa1] == 0, {sa5, sa3, sa1}];
dataC5 = Solve[eqsSsU[2.5, 2.3, sa5, sa3, sa1] == 0 && eqsSsU[2.6, 2.2, sa5, sa3, sa1] == 0 &&
     eqsSsU[2.7, 2.1, sa5, sa3, sa1] == 0, {sa5, sa3, sa1}];
dataC6 = Solve[eqsSsU[2.6, 2.4, sa5, sa3, sa1] == 0 && eqsSsU[2.7, 2.3, sa5, sa3, sa1] == 0 &&
     eqsSsU[2.8, 2.2, sa5, sa3, sa1] == 0, {sa5, sa3, sa1}];
```

```
Clear[fSa1, fSa3, fSa5];
fSa1[U_, sa10_, sa11_] := sa10 + sa11 * U;
fSa3[U_, sa30_, sa31_, sa32_, sa33_] := sa30 + sa31 * U + sa32 * U^2 + sa33 * U^3;
fSa5[U_, sa50_, sa51_, sa52_, sa53_, sa54_, sa55_] :=
  sa50 + sa51 * U + sa52 * U^2 + sa53 * U^3 + sa54 * U^4 + sa55 * U^5;
```

```
Clear[xs1, xs2];
xs1 = sa1 /. Part[Part[dataC1, 1], 3];
xs2 = sa1 /. Part[Part[dataC2, 1], 3];
```

```
In[•]:= Solve[fSa1[2.0, sa10, sa11] == xs1 && fSa1[2.1, sa10, sa11] == xs2, {sa10, sa11}]
Out[•]= {{sa10 → 1.8082 × 10^-6, sa11 → -8.93712 × 10^-7}}
```

```
Clear[ys1, ys2, ys3, ys4];
ys1 = sa3 /. Part[Part[dataC1, 1], 2];
ys2 = sa3 /. Part[Part[dataC2, 1], 2];
ys3 = sa3 /. Part[Part[dataC3, 1], 2];
ys4 = sa3 /. Part[Part[dataC4, 1], 2];
```

```
In[•]:= Solve[fSa3[2.0, sa30, sa31, sa32, sa33] == ys1 &&
  fSa3[2.1, sa30, sa31, sa32, sa33] == ys2 && fSa3[2.2, sa30, sa31, sa32, sa33] == ys3 &&
  fSa3[2.3, sa30, sa31, sa32, sa33] == ys4, {sa30, sa31, sa32, sa33}]
Out[•]= {{sa30 → 48., sa31 → -84., sa32 → 54., sa33 → -8.04724 × 10^-7}}
```

```
Clear[zs1, zs2, zs3, zs4, zs5, zs6];
zs1 = sa5 /. Part[Part[dataC1, 1], 1];
zs2 = sa5 /. Part[Part[dataC2, 1], 1];
zs3 = sa5 /. Part[Part[dataC3, 1], 1];
zs4 = sa5 /. Part[Part[dataC4, 1], 1];
zs5 = sa5 /. Part[Part[dataC5, 1], 1];
zs6 = sa5 /. Part[Part[dataC6, 1], 1];
```

```
In[•]:= Solve[fSa5[2.0, sa50, sa51, sa52, sa53, sa54, sa55] == zs1 &&
  fSa5[2.1, sa50, sa51, sa52, sa53, sa54, sa55] == zs2 &&
  fSa5[2.2, sa50, sa51, sa52, sa53, sa54, sa55] == zs3 &&
  fSa5[2.3, sa50, sa51, sa52, sa53, sa54, sa55] == zs4 &&
  fSa5[2.4, sa50, sa51, sa52, sa53, sa54, sa55] == zs5 &&
  fSa5[2.5, sa50, sa51, sa52, sa53, sa54, sa55] == zs6, {sa50, sa51, sa52, sa53, sa54, sa55}]
```

```
Out[•]= {{sa50 → -152., sa51 → 600.001,
  sa52 → -822.001, sa53 → 488., sa54 → -108., sa55 → 6.30551 × 10^-6}}
```

```
In[•]:= Clear[symSU];
symSU[U_, V_] := 2 (27 U^2 - 40 U + 10) (U - 1)^2 (2 (U - 1)^2 - U^2) +
   (-152 + 600 U - 822 U^2 + 488 U^3 - 108 U^4) V^2 + (48 - 84 U + 54 U^2) V^4;
```

4.2. The antisymmetric part of S

```
Clear[antiSsU, eqaSsU];
antiSsU[V_, sb6_, sb4_, sb2_, sb0_] := sb6 * V + sb4 * V^3 + sb2 * V^5 + sb0 * V^7;
eqaSsU[R_, S_, sb6_, sb4_, sb2_, sb0_] := antiSsU[(R - S) / 2, sb6, sb4, sb2, sb0] -
   ((twist2[{1, R, 1, 0, S, 0, 0}] - tw2Ord0[1, R, S] - finalP[R, S] / denomi22[1, R, S]) R -
      (twist2[{1, R, 0, 0, S, 1, 0}] - tw2Ord0[1, R, S] - finalP[S, R] / denomi22[1, R, S])
       S) denoS[R, S] / 2;
```

```
Clear[dataD1, dataD2, dataD3, dataD4, dataD5, dataD6, dataD7, dataD8];
dataD1 = Solve[diffCstUb[2.1, 1.9, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.2, 1.8, sb6, sb4, sb2, sb0] == 0 && eqaSsU[2.3, 1.7, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.4, 1.6, sb6, sb4, sb2, sb0] == 0, {sb6, sb4, sb2, sb0}];
dataD2 = Solve[eqaSsU[2.2, 2.0, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.3, 1.9, sb6, sb4, sb2, sb0] == 0 && eqaSsU[2.4, 1.8, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.5, 1.7, sb6, sb4, sb2, sb0] == 0, {sb6, sb4, sb2, sb0}];
dataD3 = Solve[eqaSsU[2.3, 2.1, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.4, 2.0, sb6, sb4, sb2, sb0] == 0 && eqaSsU[2.5, 1.9, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.6, 1.8, sb6, sb4, sb2, sb0] == 0, {sb6, sb4, sb2, sb0}];
dataD4 = Solve[eqaSsU[2.4, 2.2, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.5, 2.1, sb6, sb4, sb2, sb0] == 0 && eqaSsU[2.6, 2.0, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.7, 1.9, sb6, sb4, sb2, sb0] == 0, {sb6, sb4, sb2, sb0}];
dataD5 = Solve[eqaSsU[2.5, 2.3, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.6, 2.2, sb6, sb4, sb2, sb0] == 0 && eqaSsU[2.7, 2.1, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.8, 2.0, sb6, sb4, sb2, sb0] == 0, {sb6, sb4, sb2, sb0}];
dataD6 = Solve[eqaSsU[2.6, 2.4, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.7, 2.3, sb6, sb4, sb2, sb0] == 0 && eqaSsU[2.8, 2.2, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.9, 2.1, sb6, sb4, sb2, sb0] == 0, {sb6, sb4, sb2, sb0}];
dataD7 = Solve[eqaSsU[2.7, 2.5, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.8, 2.4, sb6, sb4, sb2, sb0] == 0 && eqaSsU[2.9, 2.3, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[3.0, 2.2, sb6, sb4, sb2, sb0] == 0, {sb6, sb4, sb2, sb0}];
dataD8 = Solve[eqaSsU[2.8, 2.6, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[2.9, 2.5, sb6, sb4, sb2, sb0] == 0 && eqaSsU[3.0, 2.4, sb6, sb4, sb2, sb0] == 0 &&
    eqaSsU[3.1, 2.3, sb6, sb4, sb2, sb0] == 0, {sb6, sb4, sb2, sb0}];
```

```
Clear[wt1];
wt1 = sb0 /. Part[Part[dataD2, 1], 4]
```

Out[•]= 9.08402×10^{-8}

```
Clear[fSb2, fSb4, fSb6];
fSb2[U_, sb20_, sb21_, sb22_] := sb20 + sb21 * U + sb22 * U^2;
fSb4[U_, sb40_, sb41_, sb42_, sb43_, sb44_] :=
  sb40 + sb41 * U + sb42 * U^2 + sb43 * U^3 + sb44 * U^4;
fSb6[U_, sb60_, sb61_, sb62_, sb63_, sb64_, sb65_, sb66_] :=
  sb60 + sb61 * U + sb62 * U^2 + sb63 * U^3 + sb64 * U^4 + sb65 * U^5 + sb66 * U^6;

Clear[xt1, xt2, xt3];
xt1 = sb2 /. Part[Part[dataD4, 1], 3];
xt2 = sb2 /. Part[Part[dataD2, 1], 3];
xt3 = sb2 /. Part[Part[dataD3, 1], 3];
```

In[•]:=
```
Solve[fSb2[2.3, sb20, sb21, sb22] == xt1 && fSb2[2.1, sb20, sb21, sb22] == xt2 &&
  fSb2[2.2, sb20, sb21, sb22] == xt3, {sb20, sb21, sb22}]
```

Out[•]= $\left\{\left\{\text{sb20} \to 40.,\ \text{sb21} \to -54.,\ \text{sb22} \to -6.33287 \times 10^{-7}\right\}\right\}$

```
Clear[yt1, yt2, yt3, yt4, yt5];
yt1 = sb4 /. Part[Part[dataD6, 1], 2];
yt2 = sb4 /. Part[Part[dataD2, 1], 2];
yt3 = sb4 /. Part[Part[dataD3, 1], 2];
yt4 = sb4 /. Part[Part[dataD4, 1], 2];
yt5 = sb4 /. Part[Part[dataD5, 1], 2];
```

In[•]:=
```
Solve[fSb4[2.5, sb40, sb41, sb42, sb43, sb44] == yt1 &&
  fSb4[2.1, sb40, sb41, sb42, sb43, sb44] == yt2 &&
  fSb4[2.2, sb40, sb41, sb42, sb43, sb44] == yt3 &&
  fSb4[2.3, sb40, sb41, sb42, sb43, sb44] == yt4 &&
  fSb4[2.4, sb40, sb41, sb42, sb43, sb44] == yt5, {sb40, sb41, sb42, sb43, sb44}]
```

Out[•]= $\left\{\left\{\text{sb40} \to -104.,\ \text{sb41} \to 394.,\ \text{sb42} \to -400.,\ \text{sb43} \to 108.,\ \text{sb44} \to -1.25397 \times 10^{-6}\right\}\right\}$

```
Clear[zt1, zt2, zt3, zt4, zt5, zt6, zt7];
zt1 = sb6 /. Part[Part[dataD8, 1], 1];
zt2 = sb6 /. Part[Part[dataD2, 1], 1];
zt3 = sb6 /. Part[Part[dataD3, 1], 1];
zt4 = sb6 /. Part[Part[dataD4, 1], 1];
zt5 = sb6 /. Part[Part[dataD5, 1], 1];
zt6 = sb6 /. Part[Part[dataD6, 1], 1];
zt7 = sb6 /. Part[Part[dataD7, 1], 1];
```

```
In[•]:= Solve[fSb6[2.7, sb60, sb61, sb62, sb63, sb64, sb65, sb66] == zt1 &&
          fSb6[2.1, sb60, sb61, sb62, sb63, sb64, sb65, sb66] == zt2 &&
          fSb6[2.2, sb60, sb61, sb62, sb63, sb64, sb65, sb66] == zt3 &&
          fSb6[2.3, sb60, sb61, sb62, sb63, sb64, sb65, sb66] == zt4 &&
          fSb6[2.4, sb60, sb61, sb62, sb63, sb64, sb65, sb66] == zt5 &&
          fSb6[2.5, sb60, sb61, sb62, sb63, sb64, sb65, sb66] == zt6 &&
          fSb6[2.6, sb60, sb61, sb62, sb63, sb64, sb65, sb66] == zt7,
         {sb60, sb61, sb62, sb63, sb64, sb65, sb66}]
Out[•]= {{sb60 → -0.0000213046, sb61 → -144., sb62 → 552.,
          sb63 → -714., sb64 → 360., sb65 → -54., sb66 → -1.26675 × 10^-7}}

In[•]:= Clear[antiSU];
        antiSU[U_, V_] := (-144 U + 552 U^2 - 714 U^3 + 360 U^4 - 54 U^5) V +
            (-104 + 394 U - 400 U^2 + 108 U^3) V^3 + (40 - 54 U) V^5;
        finalS[R_, S_] := symSU[(R + S) / 2, (R - S) / 2] + antiSU[(R + S) / 2, (R - S) / 2];

In[•]:= Simplify[finalS[R, S]]
Out[•]= (-1 + S)^2 (40 (-1 + S)^2 + 3 R^3 (-16 + 9 S) - 80 R (2 - 3 S + S^2) + 3 R^2 (56 - 56 S + 9 S^2))
```

5. The polynomial $T(L, R_0, R_1)$

```
        Clear[denoR4, polyT, eqpolyT];
        denoR4[L_, R_, S_] := -192 Sqrt[L (R - L) (S - L) (R + S - L)] (R - L) (R + S - L);
        polyT[L_, R_, S_, a0_, a1_, a2_, a3_, a4_] :=
          S^2 L^2 R + S (a0 * L^4 + a1 * L^3 R + a2 * L^2 R^2 + a3 * R^3 * L + a4 * R^4) +
           R ((1 - a0) * L^4 + (-2 - a1) * L^3 R - a2 * L^2 R^2 - a3 * R^3 * L - a4 * R^4);
        eqpolyT[R_, S_, a0_, a1_, a2_, a3_, a4_] := polyT[1, R, S, a0, a1, a2, a3, a4] -
           denoR4[1, R, S] (twist2[{1, R, 0, 1, S, 0, 0}] - tw2Ord0[1, R, S]);

In[•]:= Solve[eqpolyT[2.0, 1.5, a0, a1, a2, a3, a4] == 0 && eqpolyT[2.5, 1.5, a0, a1, a2, a3, a4] == 0 &&
          eqpolyT[3.0, 1.5, a0, a1, a2, a3, a4] == 0 && eqpolyT[3.5, 1.5, a0, a1, a2, a3, a4] == 0 &&
          eqpolyT[4.0, 1.5, a0, a1, a2, a3, a4] == 0, {a0, a1, a2, a3, a4}]
Out[•]= {{a0 → -1.06886 × 10^-11, a1 → -2.,
          a2 → -8.19435 × 10^-12, a3 → 1.87561 × 10^-12, a4 → -1.57367 × 10^-13}}

In[•]:= Clear[finalT];
        finalT[L_, R_, S_] := polyT[L, R, S, 0, -2, 0, 0, 0];

In[•]:= Simplify[finalT[L, R, S]]
Out[•]= L^2 R (L - S)^2
```